An practical introduction to Machine Learning

Eric Medvet

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Outline

Machine Learning: what and why? Real cases Motivating example

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Tree-based methods

Regression trees Trees aggregation Binary classification

Section 1

Machine Learning: what and why?

Subsection 1

Real cases

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Real cases

Pro-active customer care in telecommunication

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Collision detection in insurance

What is Machine Learning?

Definition Machine Learning is the science of getting computer to learn without being explicitly programmed.

In practice

A set of mathematical and statistical tools for:

 building a model which allows to predict an output, given an input (*supervised learning*)

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 learn relationships and structures in data (unsupervised learning)

Machine Learning everyday

Example problem: spam

Discriminate between spam and non-spam emails.

Google	in:spam		
Gmail -	•	C More *	
COMPOSE			Delete all spam
		CSC Conference Secretari.	Call for Papers : 1st Annual Intern
Inbox (3) Starred Important Chats Sent Mail Drafts	🗆 🛣 🔊	Alexander Horn	Recently posted academic job vac
		Regalo di Benvenuto	emedvet@units.it per te uno Smar
		Peugeot Italia	Peugeot supervaluta il tuo usato. I
		CAP petite enfance	votre profil nous intéresse - Vous r
Spam (526)	1 X D	Rachat de crédits	Réduisez vos mensualités jusqu'à
Categories Social Promotions (1) Updates (1) Purchases Travel Finance		Zalando	Le sneakers che conquistano la st
		Sondage National	Pour ou contre passer à 90 km/h s
	□ ☆ ▶	Oroscopo	Messaggio Privato per - Stai riceve
		Secret Escapes	Sconti Imbattibili su Hotel e Vacan
		Erogazione credito appro.	Fino a 50.000 euro, anche protesta

Figure: Spam filtering in Gmail.

Machine Learning everyday

Example problem: image understanding Recognize objects in images.



Figure: Object recognition in Google Photos.

Why ML/DM "today"?

- we collect more and more data (big data)
- we have more and more computational power



Figure: From http://www.mkomo.com/cost-per-gigabyte-update.

ML/DM is popular!



Figure: Popular areas of interest, from the Skill Up 2016: Developer Skills Report²

¹https://techcus.com/p/r1zSmbXut/ top-5-highest-paying-programming-languages-of-2016/. ²https://techcus.com/p/r1zSmbXut/ top-5-highest-paying-programming-languages-of-2016/.

Be able to:

- 1. design
- 2. implement
- 3. assess experimentally

an end-to-end Machine Learning or Data Mining system.

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Write some code!

Be able to:

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- 2. implement
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- Which is the problem to be solved? Which are the input and output? Which are the most suitable algorithms? How should data be prepared? Does computation time matter?
- Write some code!
- How to measure solution quality? How to compare solutions? Is my solution general?

Subsection 2

Motivating example

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The amateur botanist friend

He likes to collect Iris plants. He "realized" that there are 3 species, in particular, that he likes: *Iris setosa, Iris virginica,* and *Iris versicolor*. He'd like to have a tool to automatically *classify* collected samples in one of the 3 species.



Figure: Iris versicolor.

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How to help him?

Which is the problem to be solved?

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Which is the problem to be solved?

Assign exactly one specie to a sample.

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- Which is the problem to be solved?
 - Assign exactly one specie to a sample.

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Which are the input and output?

- Which is the problem to be solved?
 - Assign exactly one specie to a sample.
- Which are the input and output?
 - Output: one species among I. setosa, I. virginica, I. versicolor.

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Input: the plant sample...

- Which is the problem to be solved?
 - Assign exactly one specie to a sample.
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 - Output: one species among I. setosa, I. virginica, I. versicolor.

- Input: the plant sample...
 - a description in natural language?

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 - Assign exactly one specie to a sample.
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- Input: the plant sample...
 - a description in natural language?
 - a digital photo?

- Which is the problem to be solved?
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- Input: the plant sample...
 - a description in natural language?
 - a digital photo?
 - DNA sequences?

- Which is the problem to be solved?
 - Assign exactly one specie to a sample.
- Which are the input and output?
 - Output: one species among I. setosa, I. virginica, I. versicolor.

- Input: the plant sample...
 - a description in natural language?
 - a digital photo?
 - DNA sequences?
 - some measurements of the sample!

Iris: input and output



Figure: Sepal and petal.

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Input: sepal length and width, petal length and width (in cm) Output: the class Example: $(5.1, 3.5, 1.4, 0.2) \rightarrow I$. setosa

Other information

The botanist friend asked a senior botanist to inspect several samples and label them with the corresponding species.

Sepal length	Sepal width	Petal length	Petal width	Species
5.1	3.5	1.4	0.2	I. setosa
4.9	3.0	1.4	0.2	I. setosa
7.0	3.2	4.7	1.4	I. versicolor
6.0	2.2	5.0	1.5	I. virginica

 Sepal length, sepal width, petal length, and petal width are input variables (or independent variables, or features, or attributes).

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 Species is the output variable (or dependent variable, or response).

$$\mathbf{X} = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,p} \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

x₁^T = (x_{1,1}, x_{1,2},..., x_{1,p}) is an observation (or instance, or data point), composed of p variable values;

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$$\mathbf{X} = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,p} \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

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- x₁^T = (x_{1,1}, x_{1,2},..., x_{1,p}) is an observation (or instance, or data point), composed of p variable values; y₁ is the corresponding output variable value
- ▶ $\mathbf{x}_2^T = (x_{1,2}, x_{2,2}, \dots, x_{n,2})$ is the vector of all the *n* values for the 2nd variable (X₂).

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Simplification: forget petal and I. virginica \rightarrow 2 variables, 2 species (binary classification problem).



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 Problem: given any new observation, we want to automatically assign the species.



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- Sketch of a possible solution:



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- Sketch of a possible solution:
 - 1. learn a model (classifier)


Iris: visual interpretation

Simplification: forget petal and I. virginica \rightarrow 2 variables, 2 species (binary classification problem).

- Problem: given any new observation, we want to automatically assign the species.
- Sketch of a possible solution:
 - 1. learn a model (classifier)
 - 2. "use" model on new observations



Section 2

Tree-based methods

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The carousel robot attendant

Problem: replace the carousel attendant with a robot which automatically decides who can ride the carousel.



Observed human attendant's decisions.



How can the robot take the decision?

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How can the robot take the decision?

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• if younger than $10 \rightarrow can't!$

Observed human attendant's decisions.



How can the robot take the decision?

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- if younger than $10 \rightarrow can't!$
- otherwise:

Observed human attendant's decisions.



How can the robot take the decision?

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- if younger than $10 \rightarrow can't!$
- otherwise:
 - if shorter than 120 \rightarrow can't!
 - otherwise \rightarrow can!

Observed human attendant's decisions.



How can the robot take the decision?

- if younger than $10 \rightarrow can't!$
- otherwise:
 - if shorter than 120 \rightarrow can't!
 - otherwise \rightarrow can!





How to build a decision tree

Dividi-et-impera (recursively):

- find a cut variable and a cut value
- for left-branch, dividi-et-impera
- for right-branch, dividi-et-impera

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How to build a decision tree: detail

```
Recursive binary splitting
   function BUILDDECISIONTREE(X, y)
       if SHOULDSTOP(y) then
             \hat{y} \leftarrow \text{most common class in } \mathbf{y}
             return new terminal node with \hat{y}
        else
             (i, t) \leftarrow \text{BestBranch}(\mathbf{X}, \mathbf{y})
             n \leftarrow new branch node with (i, t)
             append child BUILDDECISIONTREE(\mathbf{X}|_{\mathbf{x}_i < t}, \mathbf{y}|_{\mathbf{x}_i < t}) to n
             append child BUILDDECISIONTREE(\mathbf{X}|_{\mathbf{x}_i > t}, \mathbf{y}|_{\mathbf{x}_i > t}) to n
             return n
        end if
   end function
```

- Recursive binary splitting
- Top down (start from the "big" problem)

Best branch

$$\begin{array}{l} \text{function BestBranch}(\mathbf{X}, \mathbf{y}) \\ (i^{\star}, t^{\star}) \leftarrow \arg\min_{i,t} E(\mathbf{y}|_{\mathbf{x}_i \geq t}) + E(\mathbf{y}|_{\mathbf{x}_i < t}) \\ \text{return } (i^{\star}, t^{\star}) \\ \text{end function} \end{array}$$

Classification error on subset:

$$egin{aligned} \mathsf{E}(\mathbf{y}) &= rac{|\{y \in \mathbf{y} : y
eq \hat{y}\}|}{|\mathbf{y}|} \ \hat{y} &= ext{the most common class in } \mathbf{y} \end{aligned}$$

Greedy (choose split to minimize error now, not in later steps)

Best branch

$$(i^{\star}, t^{\star}) \leftarrow \operatorname*{arg\,min}_{i,t} E(\mathbf{y}|_{\mathbf{x}_i \geq t}) + E(\mathbf{y}|_{\mathbf{x}_i < t})$$

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The formula say what is done, not how is done!

Q: different "how" can differ? how?

Stopping criterion

function SHOULDSTOP(y) if y contains only one class then return true else if $|y| < k_{min}$ then return true else return false end if end function

Other possible criterion:

tree depth larger than d_{max}

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Categorical independent variables

- Trees can work with categorical variables
- ▶ Branch node is $x_i = c$ or $x_i \in C' \subset C$ (*c* is a class)

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Can mix categorical and numeric variables

Stopping criterion: role of k_{\min}



When the tree is "too complex"

- less readable/understandable/explicable
- maybe there was noise into the data
- Q: what's noise in carousel data?

Tree complexity issue is not related (only) with k_{\min}

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Tree complexity: other interpretation

maybe there was noise into the data

The tree *fits* the learning data too much:

- it overfits (overfitting)
- does not generalize (high variance: model varies if learning data varies)

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"model varies if learning data varies": what? why data varies?

learning data is about the system/phenomenon/nature S

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- ► a collection of *observations* of *S*
- a point of view on S

"model varies if learning data varies": what? why data varies?

- learning data is about the system/phenomenon/nature S
 - ► a collection of *observations* of *S*
 - a point of view on S
- learning is about understanding/knowing/explaining S

"model varies if learning data varies": what? why data varies?

- \blacktriangleright learning data is about the system/phenomenon/nature S
 - ► a collection of *observations* of *S*
 - a point of view on S
- learning is about understanding/knowing/explaining S
 - if I change the point of view on S, my knowledge about S should remain the same!

Spotting overfitting



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Test error: error on unseen data

Spotting overfitting



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Test error: error on unseen data

k-fold cross-validation

Where can I find "unseen data"? Pretend to have it!

- 1. split learning data (**X** and **y**) in k equal slices (each of $\frac{n}{k}$ observations/elements)
- 2. for each split (i.e., each $i \in \{1,\ldots,k\}$)
 - 2.1 learn on all but k-th slice
 - 2.2 compute classification error on unseen k-th slice
- 3. average the k classification errors

In essence:

- can the learner generalize on available data?
- how the learned artifact will behave on unseen data?

k-fold cross-validation



Or with classification error rate or any other meaningful (effectiveness) measure

Q: how should data be split?

Subsection 1

Regression trees

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Regression with trees

Trees can be used for regression, instead of classification.

decision tree vs. regression tree

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Tree building: decision \rightarrow regression

function BUILDDECISIONTREE(X, y) if SHOULDSTOP(y) then $\hat{y} \leftarrow \text{most common class in } \mathbf{y}$ **return** new terminal node with \hat{y} else $(i, t) \leftarrow \text{BestBranch}(\mathbf{X}, \mathbf{y})$ $n \leftarrow$ new branch node with (i, t)append child BUILDDECISIONTREE($\mathbf{X}|_{\mathbf{x}_i < t}, \mathbf{y}|_{\mathbf{x}_i < t}$) to *n* append child BUILDDECISIONTREE($\mathbf{X}|_{\mathbf{x}_i > t}, \mathbf{y}|_{\mathbf{x}_i > t}$) to *n* return n end if end function

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Q: what should we change?

Tree building: decision \rightarrow regression

function BUILDDECISIONTREE(X, y) if SHOULDSTOP(y) then $\hat{v} \leftarrow \bar{v}$ ⊳ mean **y return** new terminal node with \hat{y} else $(i, t) \leftarrow \text{BestBranch}(\mathbf{X}, \mathbf{y})$ $n \leftarrow$ new branch node with (i, t)append child BUILDDECISIONTREE($\mathbf{X}|_{\mathbf{x}_i < t}, \mathbf{y}|_{\mathbf{x}_i < t}$) to *n* append child BUILDDECISIONTREE($\mathbf{X}|_{\mathbf{x}_i > t}, \mathbf{y}|_{\mathbf{x}_i > t}$) to *n* return n end if end function **Q:** what should we change?

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Interpretation



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Regression and overfitting



Image from F. Daolio

Trees in summary

Pros:

- ▲ easily interpretable/explicable
- ▲ learning and regression/classification easily understandable

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▲ can handle both numeric and categorical values

Cons:

▼ not so accurate (Q: always?)

Tree accuracy?



Image from An Introduction to Statistical Learning, $\langle \mathcal{P} \rangle$, $\langle \mathbb{P} \rangle$,

Subsection 2

Trees aggregation

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Weakness of the tree



Small tree:

- Iow complexity
- will hardly fit the "curve" part
- high bias, low variance

Big tree:

- high complexity
- may overfit the noise on the right part
- Iow bias, high variance

The trees view



Small tree:

 "a car is something that moves"

Big tree:

 "a car is a made-in-Germany blue object with 4 wheels, 2 doors, chromed fenders, curved rear enclosing engine"

Big tree view

A big tree:

- has a detailed view of the learning data (high complexity)
- "trusts too much" the learning data (high variance)

What if we "combine" different big tree views and ignore details on which they disagree?

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What if we "combine" different big tree views and ignore details on which they disagree?

- many views
- independent views
- aggregation of views

 \approx the wisdom of the crowds: a collective opinion may be better than a single expert's opinion

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- many views
- independent views

aggregation of views

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- many views
 - just use many trees

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independent views

aggregation of views

- many views
 - just use many trees
- independent views

aggregation of views

 just average prediction (regression) or take most common prediction (classification)

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- many views
 - just use many trees
- independent views
 - ► ??? learning is deterministic: same data ⇒ same tree ⇒ same view

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- aggregation of views
 - just average prediction (regression) or take most common prediction (classification)

Independent views \equiv different points of view \equiv different learning data

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But we have only one learning data!

Independent views: idea!

Like in cross-fold, consider only a part of the data, but:

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- instead of a subset
- a sample with repetitions

Independent views: idea!

Like in cross-fold, consider only a part of the data, but:

- instead of a subset
- ► a sample with repetitions

$$\begin{split} \mathbf{X} &= (x_1^T x_2^T x_3^T x_4^T x_5^T) & \text{original learning data} \\ \mathbf{X}_1 &= (x_1^T x_5^T x_3^T x_2^T x_5^T) & \text{sample 1} \\ \mathbf{X}_2 &= (x_4^T x_2^T x_3^T x_1^T x_1^T) & \text{sample 2} \\ \mathbf{X}_i &= \dots & \text{sample } i \end{split}$$

- (y omitted for brevity)
- learning data size is not a limitation (differently than with subset)

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Bagging of trees (*bootstrap*, more in general)

Tree bagging

When learning:

- 1. Repeat B times
 - $1.1\,$ take a sample of the learning data
 - 1.2 learn a tree (unpruned)

When predicting:

- 1. Repeat B times
 - 1.1 get a prediction from *i*th learned tree
- 2. predict the average (or most common) prediction

For classification, other aggregations can be done: majority voting (most common) is the simplest

How many trees?

B is a parameter:

when there is a parameter, there is the problem of finding a good value

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remember k_{min}, depth (Q: impact on?)

How many trees?

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- remember k_{min}, depth (Q: impact on?)
- it has been shown (experimentally) that
 - ▶ for "large" *B*, bagging is better than single tree
 - increasing B does not cause overfitting
 - (for us: default B is ok! "large" \approx hundreds)
- Q: how better? at which cost?

Bagging



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Despite being learned on different samples, bagging trees may be correlated, hence views are not very independent

 e.g., one variable is much more important than others for predicting (strong predictor)

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Idea: force point of view differentiation by "hiding" variables

Random forest

When learning:

- 1. Repeat B times
 - $1.1\,$ take a sample of the learning data
 - 1.2 consider only m on p independent variables
 - 1.3 learn a tree (unpruned)

When predicting:

- 1. Repeat B times
 - 1.1 get a prediction from *i*th learned tree
- 2. predict the average (or most common) prediction
- (observations and) variables are randomly chosen...
- ... to learn a forest of trees

Q: are missing variables a problem?

Random forest: parameter m

How to choose the value for m?

- $m = p \rightarrow bagging$
- it has been shown (experimentally) that
 - *m* does not relate with overfitting
 - $m = \sqrt{p}$ is good for classification

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- $m = \frac{p}{3}$ is good for regression
- (for us, default m is ok!)

Random forest

Experimentally shown: one of the "best" multi-purpose supervised classification methods

Manuel Fernández-Delgado et al. "Do we need hundreds of classifiers to solve real world classification problems". In: J. Mach. Learn. Res 15.1 (2014), pp. 3133–3181



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but...

No free lunch!

"Any two optimization algorithms are equivalent when their performance is averaged across all possible problems"

 David H Wolpert. "The lack of a priori distinctions between learning algorithms". In: Neural computation 8.7 (1996), pp. 1341–1390

Why free lunch?

- many restaurants, many items on menus, many possibly prices for each item: where to go to eat?
- no general answer
- but, if you are a vegan, or like pizza, then a best choice could exist
- **Q:** problem? algorithm?

Nature of the prediction

Consider classification:

- tree \rightarrow the class
- \blacktriangleright forest \rightarrow the class, as resulting from a voting

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Nature of the prediction

Consider classification:

- ▶ tree → the class
 - "virginica" is just "virginica"
- \blacktriangleright forest \rightarrow the class, as resulting from a voting
 - "241 virginica, 170 versicolor, 89 setosa" is different than "478 virginica, 10 versicolor, 2 setosa"

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Is this information useful/exploitable?

Confidence/tunability

Voting outcome:

- in classification, a measure of confidence of the decision
- in binary classification, voting threshold can be tuned to adjust bias towards one class (*sensitivity*)

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Q: in regression?

Subsection 3

Binary classification

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Consider the problem of classifying a person ('s data) as suffering or not suffering from a disease X.

- positive: an observation of "suffering" class
- negative: an observation of "not suffering" class

In other problems, positive may mean a different thing: define it!

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FPR, FNR

Given some labeled data and a classifier for the disease X problem, we can measure:

- the number of negative observations wrongly classified as positives: False Positives (FP)
- the number of positive observations wrongly classified as negatives: False Negatives (FN)

To decouple FP, FN from data size:

$$FPR = \frac{FP}{N} = \frac{FP}{FP + TN}$$
$$FNR = \frac{FN}{P} = \frac{FN}{FN + TP}$$

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Accuracy and error rate

$$\mathsf{Accuracy} = 1 - \mathsf{Error} \; \mathsf{Rate}$$

 $\mathsf{Error} \; \mathsf{Rate} = rac{\mathsf{FN} + \mathsf{FP}}{\mathsf{P} + \mathsf{N}}$

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Q: Error Rate $\stackrel{?}{=} \frac{\text{FPR} + \text{FNR}}{2}$

FPR, FNR and sensitivity

- Suppose FPR = 0.06, FNR = 0.04 with threshold set to 0.5 (default for RF)
- One could be interested in "limiting" the FNR...

Experimentally:



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Receiver operating characteristic (ROC)



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Equal error rate (EER)

ROC and comparison



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C2 is better than C1: how much?

- EER
- Area under the curve (AUC)

Bagging/RF/boosting in summary

interpretability numeric/categorical accuracy test error estimate variable importance confidence/tunability fast to learn (almost) non-parametric RF

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*: Q: how faster? when? does it matter?